Don't Blink: Base Rate Neglect and the Law of Small Numbers

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This article, the fourth in a series on effective decision making, looks at factors that affect decisions.

When required to make a decision regarding the likelihood of an event occurring, our intuition (referred to as System 1 thinking in this column) often enables us to do so quickly and effortlessly. To illustrate, quickly answer the question posed in the following scenario.

Scenario 1: Karen was a highly successful athlete in high school and college and continues to compete in several team and individual sports. Karen also follows professional sports and can recite team statistics with remarkable accuracy. If you were to bet $1,000, which of the following descriptions is more likely?

a. Karen is a high school physical education (PE) teacher and coach.
b. Karen is an elementary school teacher.

We often violate simple rules of probability when making judgments about uncertain events. For example, if you are like most people, you probably guessed that Karen is a high school PE teacher and coach.

Being an athlete and avid sports fan is certainly a trait of many high school PE teachers who are coaches. However, the likelihood that Karen is a high school PE teacher and coach rather than an elementary school teacher depends on the prevalence of each teacher type. There are far more elementary school teachers than high school PE teachers, and still more relative to high school PE teachers who are also coaches.

We modeled this scenario after a now-famous decision analysis problem devised by Kahneman and Tversky (1973) that illustrates a decision-making bias called *base rate neglect*. Base rate neglect is a common decision-making error that results because we tend to ignore or to underweight base rate probabilities (i.e., the proportion of elementary school teachers to high school PE teachers who are coaches) and to rely heavily on descriptive, individuating information (i.e., athlete and avid fan).

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Scenario 2: A district has two high schools. In the larger high school, about 45 students are absent each day because of illness; in the smaller high school, about 15 students are absent each day because of illness. Assume that national reports indicate that females account for 50% of school absences due to illness daily, though the exact percentage varies from day to day.

For one year, each school recorded the days on which more than 60% of the absences due to illness were females. Which school would record more such days?

a. The larger school
b. The smaller school
c. About the same

Scenario 2—also modeled after a decision analysis problem devised by Tversky and Kahneman (1974)—reports two samples drawn from a population. The smaller school provides one sample of absences drawn from the national population of school absences, and the larger school provides a second sample.

The law of large numbers is a statistical principle that dictates that the larger the sample drawn from a population, the more likely the sample will resemble the population. Conversely, the smaller the sample,

the less likely it will resemble the population. Because the smaller district offers a smaller sample of daily absences, it is more likely to report female absentee rates that vary substantially from the population average of 50%.

The Representative Heuristic

Tversky and Kahneman find that individuals are generally insensitive to sample size when predicting the probability of events, a common decision-making bias they refer to as the law of small numbers, a tongue-in-cheek play on the law of large numbers principle (Kahneman 2011).

We recently had the opportunity to assist a district by revealing that it had fallen prey to the law of small numbers. The district in question was a rural district with a total enrollment of 1,100 students, approximately 200 of whom were eligible for special-education services (18%). The school board, concerned about funding, was pressuring administrators to attain classification rates comparable to the national average, which approximated 13%. In response, the district commissioned a group to investigate whether it was overclassifying students as a result of policy or practice. The study failed to reveal any systematic classification issues.

One of the consultants told us that the district was perplexed about why its classification rate differed substantially from the national average and decided to take a look. We recognized rather quickly that the law of small numbers was in play. District officials failed to account for sample size and erroneously believed that their classification rate (which is derived from a very small sample) should closely match the national average (i.e., the population).

Base rate neglect and the law of small numbers are components of what Tversky and Kahneman call the representativeness heuristic (Kahneman 2011). We use the representativeness heuristic when we are confronted by questions of probability; however, we tend not to think like statisticians. Instead, we engage in System 1 thinking—we rely on our intuition.

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In Scenario 1, we described Karen as a stereotypical PE teacher. So our intuition tells us that she is less likely to be an elementary school teacher; although, upon reflection, we clearly understand that there are many more elementary school teachers than high school PE teachers. Because our intuition also doesn't distinguish between large and small samples, we expect even small samples to reflect broad-based averages.

The Law of Small Numbers

Between these two common decision-making errors, we find the law of small numbers particularly pernicious. Each of us is prone to invoke the law of small numbers. Who among us has not acted on a perceived problem after hearing from a small number of teachers, students, or parents, perhaps at a board meeting or in response to a community or staff survey with a very low return rate? Moreover, the effects of this error complicate an already-complex policy environment in which school administrators make decisions.

State and local officials often praise or deride administrators and teachers when district-based, school-based, or classroom-based measures exceed or fall below statewide averages. Those officials believe incorrectly that small samples (e.g., average elementary standardized test scores in a small school) resemble the population (e.g., the average of all elementary student test scores across the state), or that small and large samples are equally representative (e.g., a smaller district and a larger district).

Overcoming the Bias

Despite the pervasiveness of this decision-making trap, administrators are far from helpless in recognizing and overcoming it. The debiasing strategy to avoid this trap is not to become statisticians; instead, recognize that your intuitive System 1 thinking is vulnerable to this common decision-making error, and shift to System 2, your reflective and analytical thinking mode.

We encourage those interested in learning more about these and other common decision-making errors to read Daniel Kahneman's bestseller *Thinking, Fast and Slow*.

References

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